Executive Summary

The internet is many things; one thing it is not: simple. There is a myriad of factors that go into the future of the internet. In the following problems, we aimed to create models for how the cost of bandwidth will change over time, for how much bandwidth will be used, and for the distribution of bandwidth.

For question one we determined that the increase of internet speed over time was a major factor in how the cost would change. We decided that using graphs helps visualize our steps to obtaining our model. We graphed the data of the average speed every year for the US and the UK then calculated the rate of change of the speed graph by finding the derivative. We graphed our model for the cost of bandwidth units over the next 10 years. Lastly, we took our final model and calculated the average cost for 2031, and compared it to the cost for 2021 to find that the cost of broadband internet decreased in ten years by approximately ten dollars.

For question two we calculated the maximum amount of bandwidth that could have been used and then we calculated the probability that all the members of each group would be operating at maximum personal bandwidth and any given time. This allowed us to determine the minimum bandwidth to meet the needs of any group by finding the expected value of specific scenarios. We demonstrated how our model functions with the three example households.

We came into this challenge with very little knowledge in the realm of modeling. In our 14 hours of work, we learned that this field of study is vast and complex, and we have a lot of respect for all of the teams that prepared and studied for the competition. We are disappointed that we were unable to complete the entirety of the challenge, but the experience of working through the problems was still very valuable in itself.

Contents

1 Introduction

The usage of high-speed internet has been increasing steadily over the last few decades, and even more so due to the effects of the pandemic which prohibits people from participating in in-person education, work, and activities. Because of this, we have come to rely on using video calls for school and work. Unfortunately, many factors of life make it difficult for people to have quality connectivity within their homes, whether it be from lack of funds or location. Because many people rely on the internet for their mobile devices to do work, broadband internet such as 5G is useful for our current technology because of its high frequencies and bandwidth. This paper outlines important questions that predict the cost, usage, and probability of online bandwidth for average consumers and households using a plethora of data and mathematical models.

2 Part 1: The Cost of Connectivity

This problem asks us to find the cost of units of bandwidth (megabits per second - Mbps) over the next 10 years for consumers in the United States and the United Kingdom.

2.1 Assumptions

- The changes within the pandemic will affect internet usage in the UK and US in the same way.

Justification: Developed nations will soon benefit from herd immunity and in a few years the effects of the pandemic will become less apparent [4]. Since the UK and US are both highly developed, not only will the pandemic improve, technology will improve at a similar rate in both countries. These increasing trends are supported in our graphs of speed per price in the UK and US, where both are exponentially increasing.

- The rate of change of download speed over time affects the cost of megabits per second of bandwidth.

Justification: From the data given in spreadsheet D2, we determined that the rate at which the speed changed over time was not constant. Thus we would need to take how the speed changed into account.

- The rate of change of the average and the average download speed has an indirect relationship with the cost of megabits of data per second.

Justification: In spreadsheet D2, for the U.S., it is shown that in 2012 the speed was much slower on average than in 2020. The average price per Mbps was also less in 2020 from 2012. From this, we concluded that as speed increases over time, cost decreases.

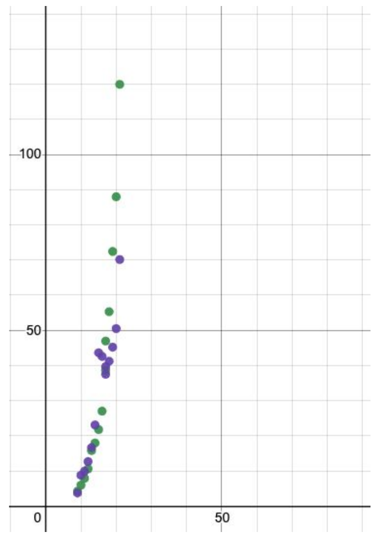
- The average downloading speed will continue to increase over the next 10 years.

Justification: High-speed internet technology is constantly being developed. For example, the development and release of 6G internet is currently being predicted for the future [7].

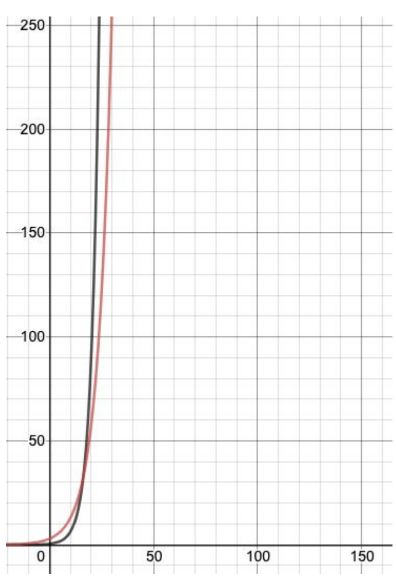
2.2 Developing the Model

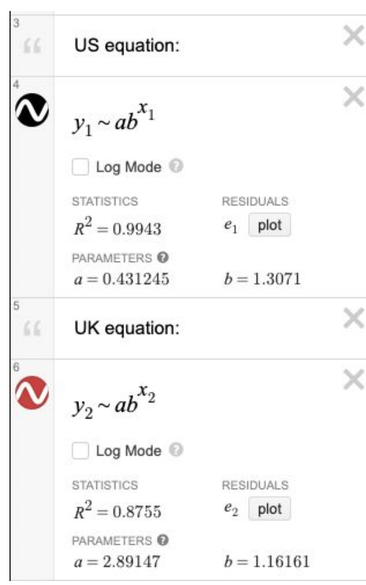
We began our model off of our assumption of the indirect relationship between download speed of the internet and cost over time. We determined the change of average download speed over time by calculating the mean of the values from spreadsheet D1 for the US and the UK from 2009 to 2020.

We then plotted the points of yearly internet speed averages from 2009 to 2020 for the U.S. and the UK as shown in the graphs below. The x-axis for both graphs represents years after the year 2000 and the y-axis represents megabits per second (Mbps). The green points represent U.S. data and the purple points represent UK data.

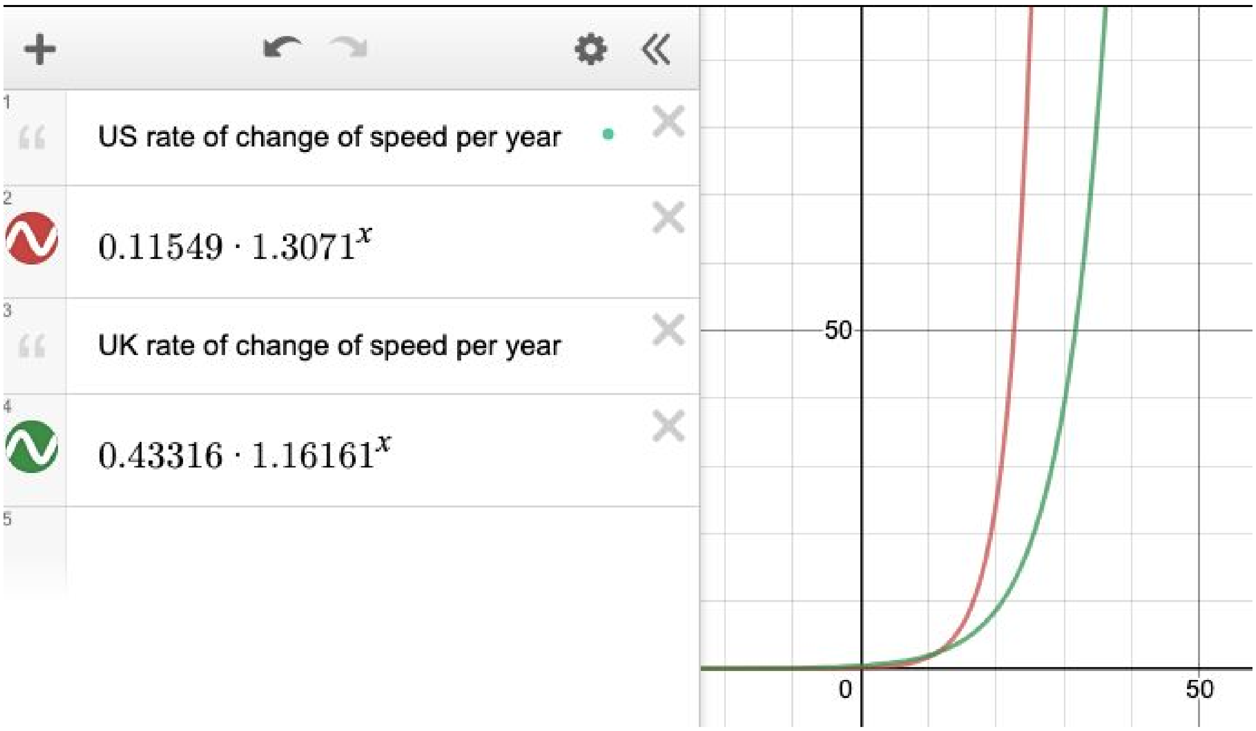


From these points, we did a regression analysis and produced the following equations for the internet speed each year with black representing the U.S. and red representing the UK. From the graph, we can clearly see that the speed of the internet is increasing exponentially over time.



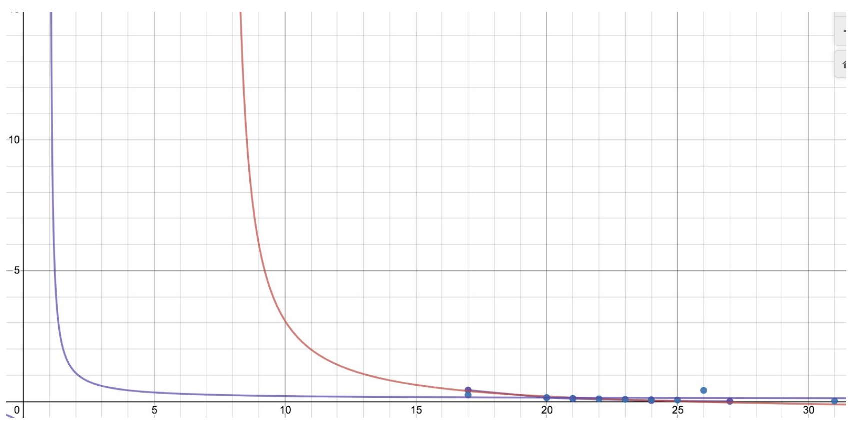


In order to see how the internet speed over time affects the cost of bandwidth, we found the derivatives of the above equations as shown below. In these graphs, the x-axis still represents time in years while the y-axis represents the rate of change of the speed. The red curve represents the rate of change of the speed in the U.S. while the green curve represents the rate of change of the speed in the UK.



From here, we used data we found from the ISP review [8] and the cost of connectivity report [9] to create graphs that showed how cost is affected by the rate of the change of the speed. To do so, we found and used the outputs from the derivative as inputs in a new function that corresponded to y-output values, representing cost of megabits per second. The inputs and outputs corresponded to specific years, which led us to know which inputs and outputs corresponded to each other.

After the creation of these new graphs, and once we knew how the rate of change affected the cost, we started two graphs for the U.S. and UK to show how the cost changed over time. We used the x-values 9-31 as inputs representing years 2009-2031. We then transferred the outputs from the rate of change to cost graphs, over to the cost per year graphs. We did this by using our tables that showed the rates of change of the speed that corresponded to each year. We knew when the rate of changes that corresponded to each year were input values, the outputs (cost) would correspond to the year that corresponded to the rate of change for that year. Finally, by plugging in the corresponding cost-year output values to the years 2009-2031 for the U.S. and UK graphs, we had our models.



Our final graph shows that as the years from 2021 increase, the cost of broadband internet in the US and UK decreases. In the year 2031, the cost per Mbps is 0.11. Comparing this to the monthly cost of internet in 2021 which is $50, and dividing that by the Mbps which is .14, we get approximately 357 megabits. Taking the Mbps for 2031 and multiplying it by 357 megabits, we get approximately $40. This means that in ten years, the average cost of mbps decreases by $10.

2.3 Discussion

With more time we would have assessed the other factors that come into play including the usage of the internet and how this is affected by the pandemic, the amount of lower class people unable to afford internet, and alternatively the amount of operators that provide relief for people in need of internet access. If we did and created models for those variables our final model would most likely have increased the cost difference from 2021-2031 because the relief packages would create a lower average cost. We would have also taken into account a cost in which companies would have stopped the decrease in cost.

3 Part 2: Bit by Bit

This problem asks us to create a model that can calculate a household's bandwidth for internet usage based on certain online activities and variables related to age range, occupation, and education. Then by using the model, calculate the minimum required bandwidth for 90% and 99% of the time.

3.1 Assumptions

- Assuming all the online activities are being pursued at the same time in each household, we can find the probability that the bandwidth will be maxed out

Justification:. By assuming this, it will allow us to calculate the minimum required bandwidth for the internet usage 90% and 99% of the time. Bandwidth represents the amount of megabits per second being used at the same time[5], therefore calculating specific hours per person isn't necessary. So if we calculate the probability of multiple people using the internet at the same time, we must use a table outlining general online activities that people of certain ages and occupations indulge in.

- We are assuming that according to the data with ranges of mbps[D5] that each activity will use the maximum number of mbps to calculate the maximum bandwidth.

Justification: To find the minimum required bandwidth we must calculate the maximum and find the probability that 90% and 99% of the time there is enough bandwidth.

- Contextual assumptions based on occupations and shift in media consumption.

Online students and online workers of that same education level have similar amounts of mbps because they indulge in the same online activities and generally use the internet at the same time. Secondly, we are excluding the use of cable TV from the data in our model because the consumption of media from TV has significantly decreased as the popularity of social media and video streaming increase [6].

- The model works for any year past the pandemic where online learning and work is prevalent, and assuming that the pandemic will virtually be over by 2024[4], this model will work for any time span of a year between 2021-2024.

3.2 Developing the Model

The minimum bandwidth required to meet the needs of a given group must be greater than the maximum possible bandwidth that the group might use. To create a flexible model to determine the minimum bandwidth, we have to calculate the probability that all members of the group will be operating at maximum capacity at any given time in a day. First we calculate the average maximum possible bandwidth and total time spent online by each individual based on their age and occupation:

Megabits used:	Before School (0-4)	School (5-17)	School (18-34)	School and Online Worker (18-34)	Online Worker (18-64)	Unemployed (18-64)	Retiree (65+)
Gaming	3	3	3	3	3	3	
Video Conferencing		4	4	4	4	4	
Video Streaming	25	25	25	25	25	25	25
Web Surfing, Email, Social Media		1	1	1	1	1	1
Large File Downloads			50	50	50	50	
Maximum Possible Bandwidth	28	33	83	83	83	83	26
Total Hours per Day	2.2304	6.453333333	11.61828571	11.61828571	11.18095238	11.18095238	7.801
Probability	0.09293333333	0.2688888889	0.4840952379	0.4840952379	0.4658730158	0.4658730158	0.3250416667

The first six rows of numbers are based on the data from spreadsheet D5. The second to last row of numbers is based on the data from spreadsheet D4, as well as references [2] and [3], with the additional assumption that all online hours per day will increase by 60% since the pandemic. The final row is calculated by total hours per day divided by 24 hours. The above information will be used to calculate the minimum necessary bandwidth for any given household group.

	Individual probability	Individual bandwidth	Expected value:
Household #1			
1: Teacher, 30s	0.4658730158	83	38.66746031
2: Unemployed, 30s	0.4658730158	83	38.66746031
3: 3 year old	0.09293333333	28	2.602133333
Group totals:	0.02017003384	194	79.93705396
Household #2			
1: Retiree, 70s	0.3250416667	26	8.451083334
2: School, 5-17	0.07682539683	33	2.535238095
3: School, 5-17	0.07682539683	33	2.535238095
Group totals:	0.001918441942	92	13.52155952
Household #3			
1: Student/online worker, 18-64	0.4840952379	83	40.17990475
2: Student/online worker, 18-64	0.4840952379	83	40.17990475
3: Student/online worker, 18-64	0.4840952379	83	40.17990475
Group totals:	0.1134468473	249	120.5397142

The above table calculates the expected values of groups based on the individuals' probabilities of using their maximums and an individuals' maximum bandwidths. The expected values are highlighted in the bottom right corners. Note that in Household #2, the maximum probabilities of the two school-age children must be multiplied by 2/7 since they only stay at the grandmother's house 2 days per week.

Also calculated in the table above are the maximum possible bandwidth values in the case that all the individuals in the group are operating at their own personal maximum bandwidth, found by summing personal bandwidths. It is highlighted in blue. The probability that this bandwidth point will ever be met is extremely low, as can be seen with the probability marked in green. Therefore, the minimum values that should support each group are the expected values marked in pink.

3.3 Sensitivity Analysis

This model best illustrates the internet usage during the pandemic because it takes into account the roles of online students and workers. If the pandemic was not relevant to our problem, this model could still be used, although data regarding total time spent on the internet should be divided by 1.6 to account for the 60% internet increase during the pandemic [1].

4 Part 3: Mobilizing Mobile

This problem asks us to develop a model that produces an optimal plan for distributing/placing cellular nodes in a region, incorporating population and demographic data for the region and taking into account the bandwidth needs of the region. We didn't get to this problem in the given time. If we had, we would have started by listing and researching factors such as demographics and location and compiling assumptions such as where the most optimal areas to place nodes in a region to most accurately fit our model

5 Conclusion

Modeling is a complex discipline. We came into the M3 Challenge with nowhere near enough knowledge and practice in modeling to do our best work. Nevertheless, working through these problems was a valuable experience for us because we saw how dynamic models can be, how many factors can affect them, and how challenging it can be to translate concepts into mathematics.

6 References

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[2]https://www.commonsensemedia.org/sites/default/files/uploads/research/census_researchreport.pdf

[3]https://ijbnpa.biomedcentral.com/articles/10.1186/1479-5868-10-102#Bib1

[4]https://www.theguardian.com/society/2021/jan/27/most-poor-nations-will-take-until-2024-to-achieve-mass-covid-19-immunisation

[5]https://www.verizon.com/info/definitions/bandwidth/

[6]https://www.marketingcharts.com/featured-105414

[7]https://www.lifewire.com/6g-wireless-4685524

[8]https://www.ispreview.co.uk/index.php/2017/05/average-cost-per-mbps-uk-fixed-broadband-speed-0-34-vs-0-85-globally.html

[9]https://www.newamerica.org/oti/reports/cost-connectivity-2020/global-findings